

# JOINT INSTITUTE FOR NUCLEAR RESEARCH



**Bogoliubov Laboratory of  
Theoretical Physics**

## ***Ground state correlations in odd-A nuclei***

S. Mishev and V.V. Voronov

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# Outline

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## **Even-even nuclei**

Theory

Occupation numbers

Transition probabilities

## **Odd-even nuclei**

Theory

Renormalized interaction strengths

Transition probabilities and spectroscopic factors

# Hamiltonian

$$H = \sum_{\tau}^{(n,p)} \left\{ \sum_{jm} (E_j - \lambda_{\tau}) a_{jm}^{\dagger} a_{jm} - \frac{1}{4} G_{\tau}^{(0)} : (P_0^{\dagger} P_0)^{\tau} : \right. \\ \left. - \frac{1}{2} \sum_{\lambda\mu} \kappa_{\lambda} : (M_{\lambda\mu}^{\dagger} M_{\lambda\mu}) : \right\}$$

$$\begin{aligned}
H = & \sum_{\tau}^{(n,p)} \left\{ \sum_{jm} (E_j - \lambda_{\tau}) a_{jm}^{\dagger} a_{jm} - \frac{1}{4} G_{\tau}^{(0)} : (P_0^{\dagger} P_0)^{\tau} : \right. \\
& \left. - \frac{1}{2} \sum_{\lambda\mu} \kappa_{\lambda} : (M_{\lambda\mu}^{\dagger} M_{\lambda\mu}) : \right\}
\end{aligned}$$

$$\alpha_{jm} = u_j a_{jm} - (-)^{j-m} v_j a_{j-m}^{\dagger}$$

$$\alpha_{jm} |qp\rangle = 0$$

$$\frac{1}{2} \sum_j (2j+1) \left\{ 1 - \frac{(E_j - \lambda)}{\sqrt{(E_j - \lambda)^2 + \Delta^2}} \right\} = n$$

$$\frac{G}{4} \sum_j \frac{2j+1}{\sqrt{(E_j - \lambda)^2 + \Delta^2}} = 1$$

# Quasiparticles and Phonons

$$H = \sum_{\tau}^{(n,p)} \left\{ \sum_{jm} \varepsilon_j \alpha_{jm}^{\dagger} \alpha_{jm} - \frac{1}{2} \sum_{\lambda\mu} \kappa_{\lambda} : (M_{\lambda\mu}^{\dagger} M_{\lambda\mu}) : \right\}$$

$$A^{\dagger}(jj'|\lambda\mu) = \sum_{mm'} \langle jmj'm'|\lambda\mu\rangle \alpha_{jm}^{\dagger} \alpha_{j'm'}^{\dagger}$$

$$Q_{\lambda\mu i}^{\dagger} = \frac{1}{2} \sum_{jj'} [\psi_{jj'}^{\lambda i} A^{\dagger}(jj'; \lambda\mu) - (-1)^{\lambda-\mu} \varphi_{jj'}^{\lambda i} A(jj'; \lambda - \mu)]$$

$$Q_{\lambda\mu i} |ph\rangle = 0$$

# EQPM-1

In RPA

$$\langle |\sum_m \alpha_{jm}^\dagger \alpha_{jm}| \rangle \approx 0$$

$$\frac{\kappa_\lambda}{2\lambda + 1} \sum_{jj'} \frac{(f_{jj'}^\lambda u_{jj'}^+)^2 (\epsilon_j + \epsilon_{j'})}{(\epsilon_j + \epsilon_{j'})^2 - \omega_{\lambda i}^2} = 1$$

In ERPA

$$\rho_j = \frac{1}{\sqrt{2j + 1}} \sum_m \langle |\alpha_{jm}^\dagger \alpha_{jm}| \rangle$$

K. Hara, *Progr. Theor. Phys.* 32, 88 (1964)

In RPA

$$\langle [Q_{\lambda\mu i}, Q_{\lambda'\mu' i'}^\dagger] \rangle = \frac{1}{2} \delta_{\lambda\lambda'} \delta_{\mu\mu'} \sum_{jj'} (\psi_{jj'}^{\lambda i} \psi_{jj'}^{\lambda i'} - \varphi_{jj'}^{\lambda i} \varphi_{jj'}^{\lambda i'})$$

In ERPA

$$\begin{aligned} \langle [Q_{\lambda\mu i}, Q_{\lambda'\mu' i'}^\dagger] \rangle &= \\ &= \frac{1}{2} \delta_{\lambda\lambda'} \delta_{\mu\mu'} \sum_{jj'} (1 - \rho_{jj'}) (\psi_{jj'}^{\lambda i} \psi_{jj'}^{\lambda i'} - \varphi_{jj'}^{\lambda i} \varphi_{jj'}^{\lambda i'}) \end{aligned}$$

$$\rho_{jj'} = \rho_j + \rho_{j'}$$

$$\frac{1}{2} \sum_j (2j+1) \left\{ 1 - \frac{(1-2\rho_j)(E_j - \lambda)}{\sqrt{(E_j - \lambda)^2 + \Delta^2}} \right\} = n$$

$$\frac{G}{4} \sum_j \frac{2j+1}{\sqrt{(E_j - \lambda)^2 + \Delta^2}} (1-2\rho_j) = 1$$

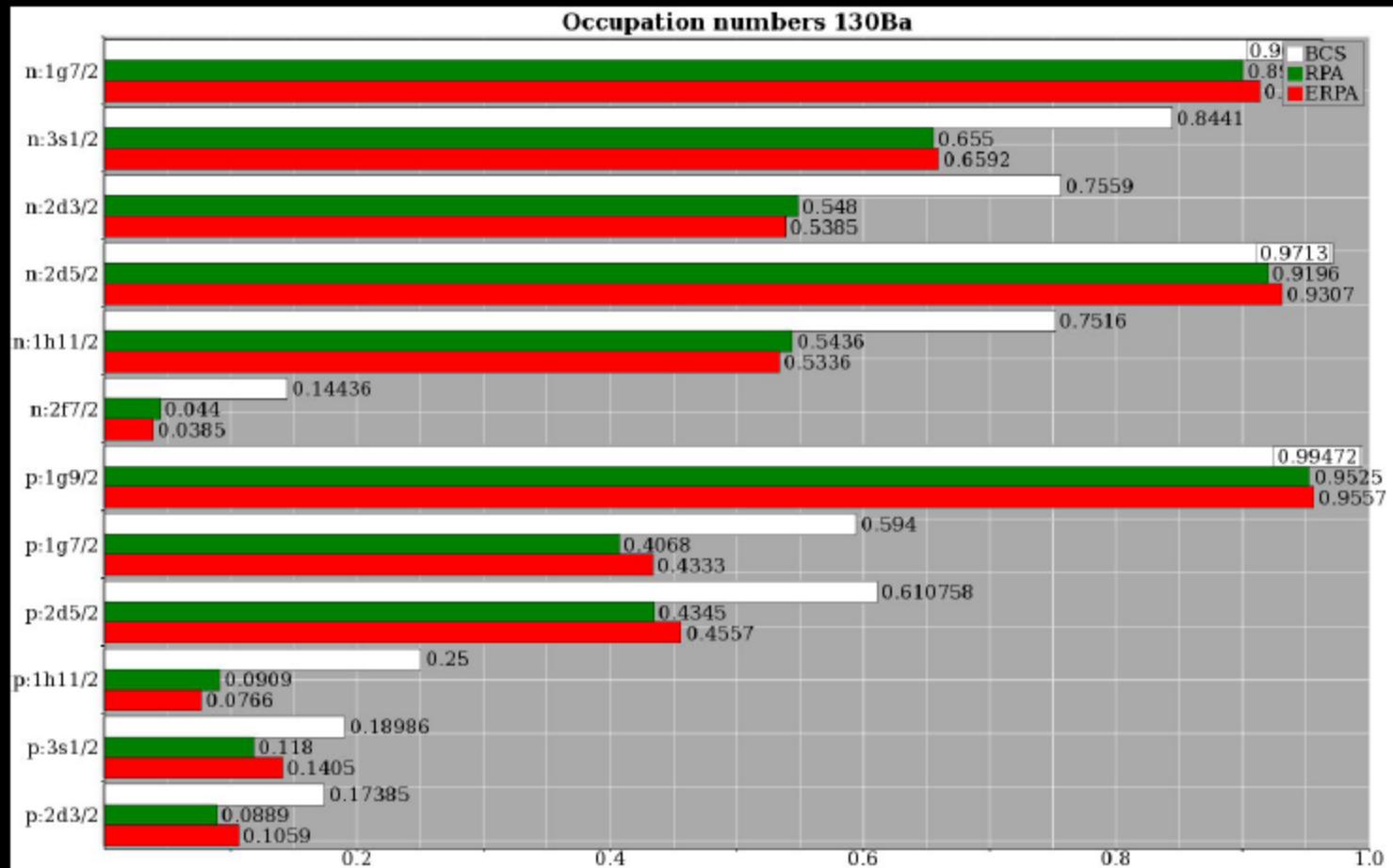
$$\frac{\kappa_\lambda}{2\lambda+1} \sum_{jj'} (1-\rho_{jj'}) \frac{(f_{jj'}^\lambda u_{jj'}^+)^2 (\varepsilon_j + \varepsilon_{j'})}{(\varepsilon_j + \varepsilon_{j'})^2 - \omega_{\lambda i}^2} = 1$$

$$\sum_{jj'} (1-\rho_{jj'}) [(\psi_{jj'}^{\lambda i})^2 - (\varphi_{jj'}^{\lambda i})^2] = 2$$

$$\rho_j = \frac{1}{2} \sum_{\lambda i j'} \frac{2\lambda+1}{2j+1} (1-\rho_{jj'}) (\varphi_{jj'}^{\lambda i})^2$$

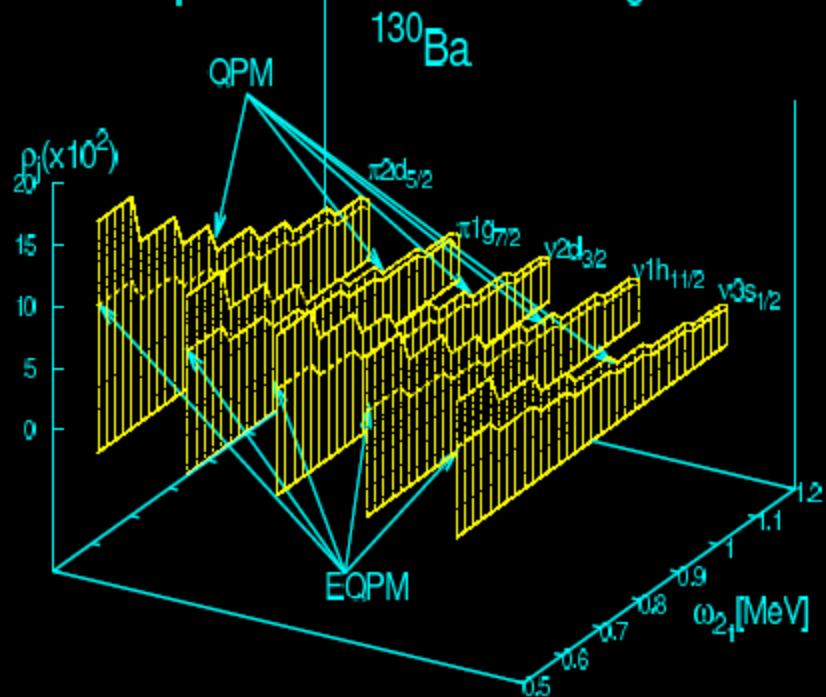


# Occupation numbers within BCS, QRPA, EQRPA

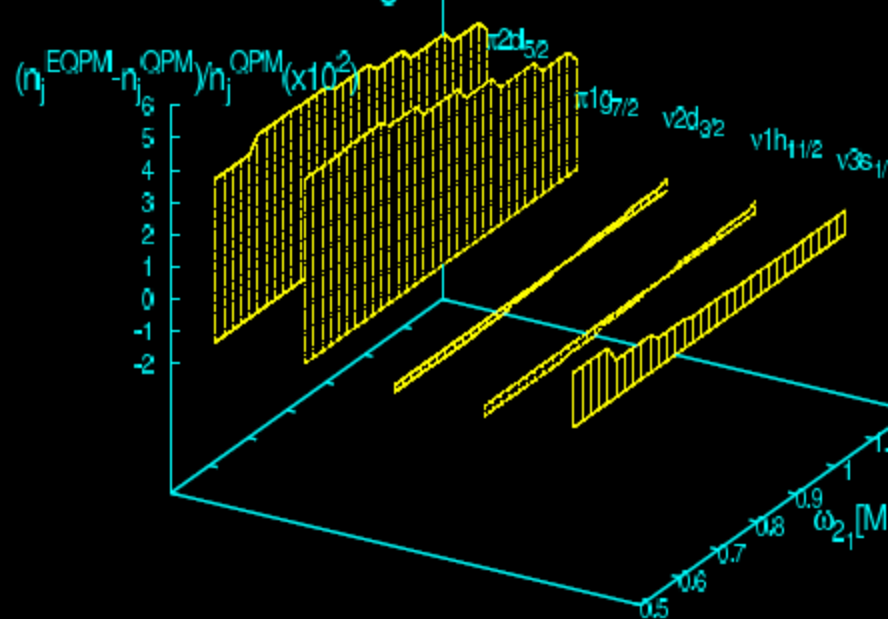


# Occupation numbers

Quasi-particle distribution in the ground state of  $^{130}\text{Ba}$

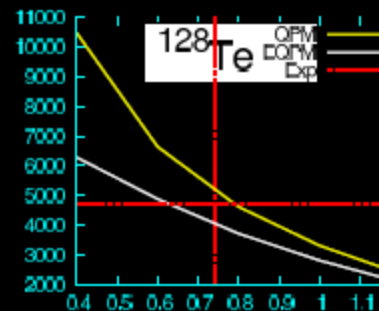
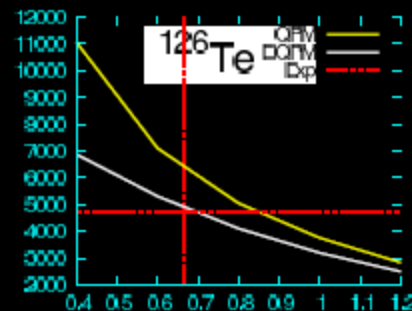
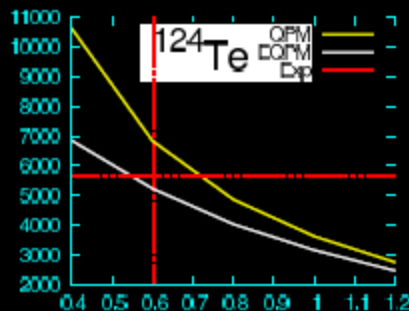
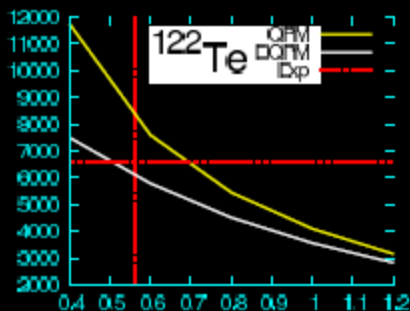
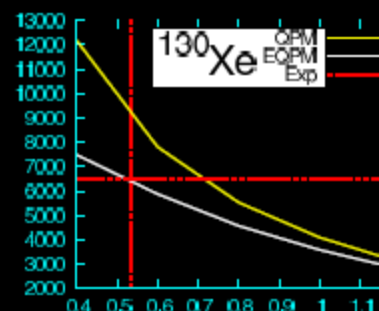
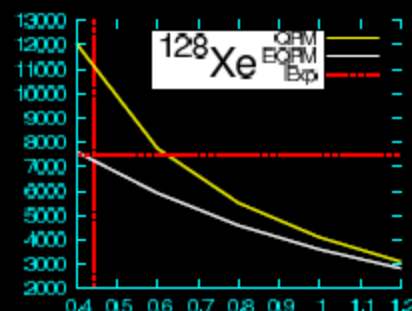
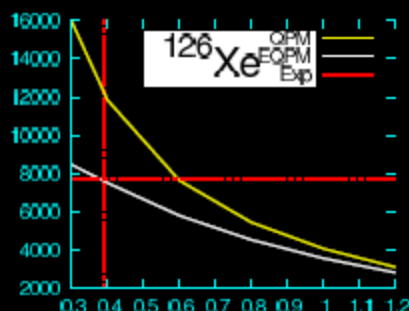
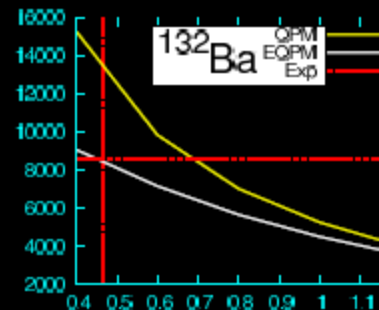
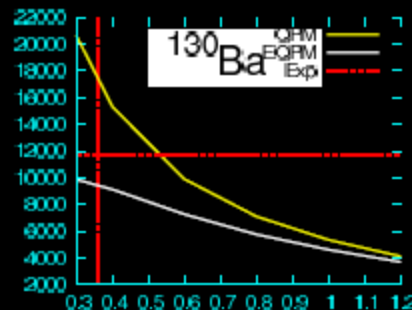


Mean number of particle deviations  $(n_j^{\text{EQPM}} - n_j^{\text{QPM}}) / n_j^{\text{QPM}}$  in the ground state of  $^{130}\text{Ba}$



# Transition probabilities in even-even nuclei

$B(E2|gs \rightarrow 2_1^+)$



$$B(E\lambda|g.s. \rightarrow \lambda_i) = \left[ \frac{1}{2} \sum (1 - \rho_{jj'}) f_{jj'}^\lambda u_{jj'}^+ g_{jj'}^{\lambda_i} \right]^2$$

# Odd-even nuclei

$$\Psi_\nu(JM) = C_{J\nu} \alpha_{JM}^\dagger + \sum_{j\lambda i} D_j^{\lambda i}(J\nu) P_{j\lambda i}^\dagger(JM) - E_{J\nu} \tilde{\alpha}_{JM} - \sum_{j\lambda i} F_j^{\lambda i}(J\nu) \tilde{P}_{j\lambda i}(JM)$$

$$P_{j\lambda i}^\dagger(JM) = [\alpha_j^\dagger Q_{\lambda i}^\dagger]_{JM}$$

$$\begin{pmatrix} \varepsilon_J & V(Jj'\lambda'i') & 0 & -W(Jj'\lambda'i') \\ V(Jj\lambda i) & K_J(j\lambda i|j'\lambda i') & W(Jj\lambda i) & 0 \\ 0 & W(Jj'\lambda'i') & -\varepsilon_J & -V(Jj'\lambda'i') \\ -W(Jj\lambda i) & 0 & -V(Jj\lambda i) & -K_J(j\lambda i|j'\lambda i') \end{pmatrix} \begin{pmatrix} C_{J\nu} \\ D_{j'\lambda'i'}(J\nu) \\ -E_{J\nu} \\ -F_{j'\lambda'i'}(J\nu) \end{pmatrix}$$

$$= \eta_{J\nu} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 - \mathcal{L}^*(Jj\lambda i) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 - \mathcal{L}^*(Jj\lambda i) \end{pmatrix} \begin{pmatrix} C_{J\nu} \\ D_{j'\lambda'i'}(J\nu) \\ -E_{J\nu} \\ -F_{j'\lambda'i'}(J\nu) \end{pmatrix}$$

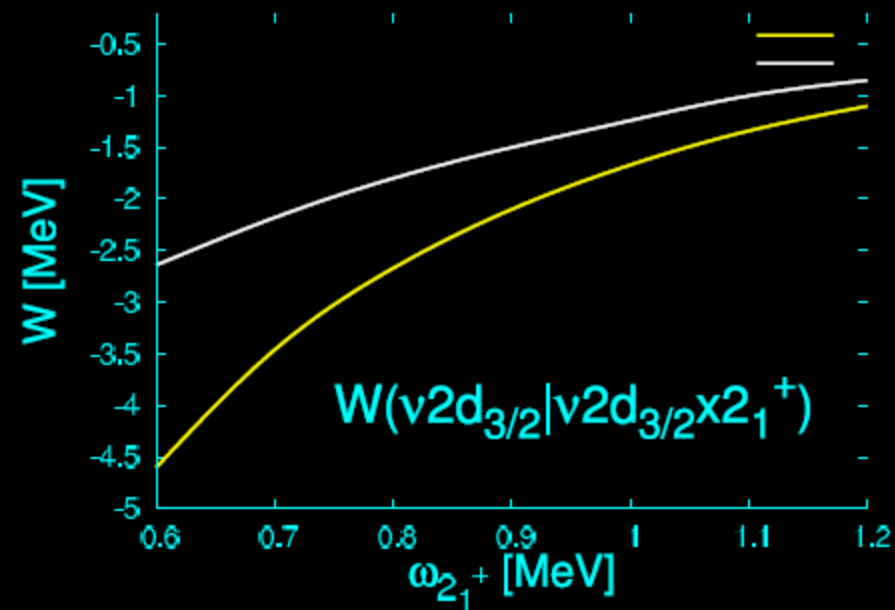
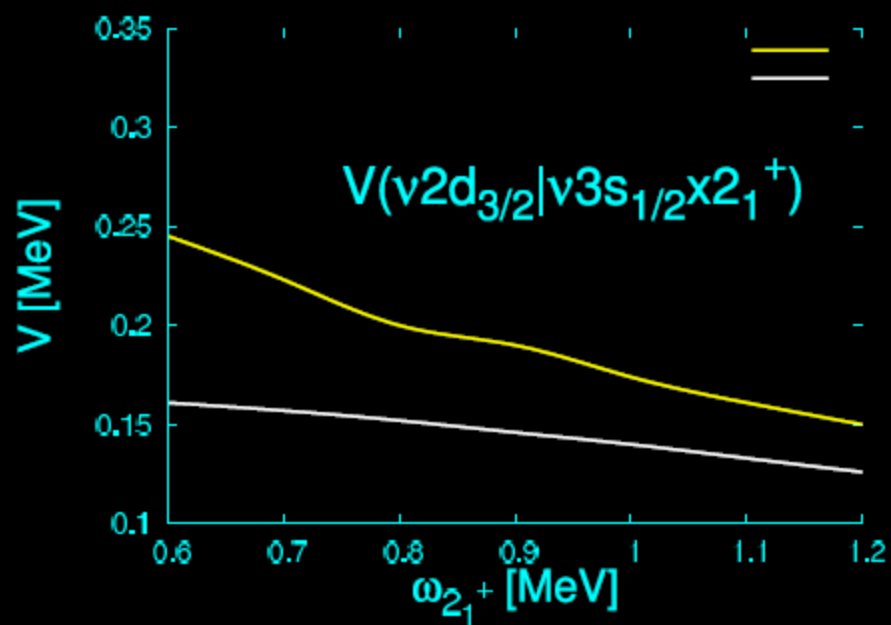
$$\begin{aligned}
V(Jj\lambda i) &= \langle |\{[\alpha_{JM}, H], P_{j\lambda i}^\dagger\} | \rangle = \\
&= -\frac{1}{\sqrt{2}}[1 - \rho_j + \mathcal{L}^*(Jj\lambda i)]\Gamma(Jj\lambda i),
\end{aligned}$$

$$\begin{aligned}
W(Jj\lambda i) &= \langle |\{[\alpha_{JM}^\dagger, H], \tilde{P}_{j\lambda i}^\dagger\} | \rangle = \\
&= \frac{\pi_\lambda}{\pi_J}\varepsilon_J\rho_j\varphi_{Jj}^{\lambda i} - \frac{1}{4}[1 - \rho_j + \mathcal{L}^*(Jj\lambda i)]\frac{\pi_\lambda}{\pi_J}\sum_{i_1}\mathcal{A}(\lambda i_1 i)\varphi_{Jj}^{\lambda i_1},
\end{aligned}$$

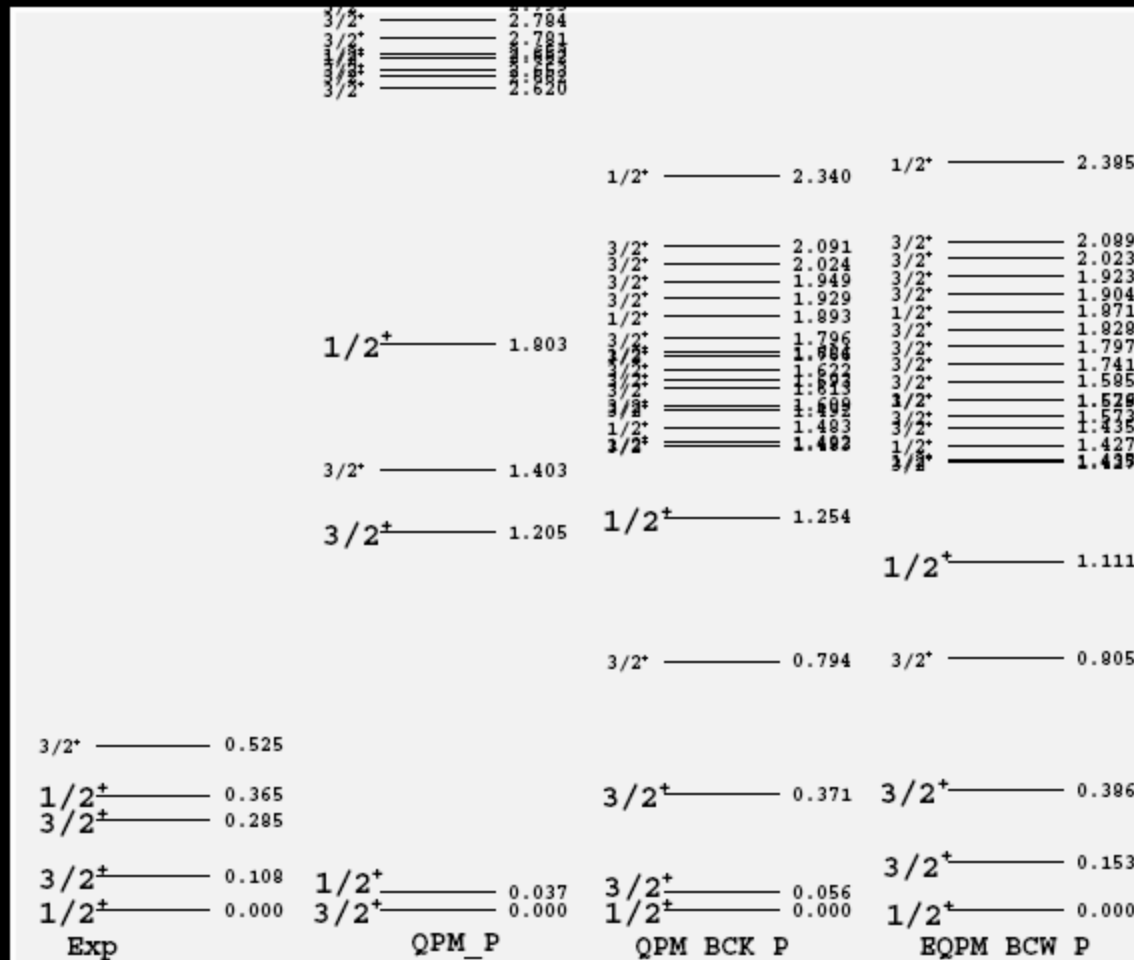
$$\begin{aligned}
K_J(j\lambda i|j'\lambda' i') &= \frac{1}{2}[I_J(j\lambda i|j'\lambda' i') + I_J(j'\lambda' i'|j\lambda i)] = \\
&= \delta_{jj'}\delta_{\lambda\lambda'}\delta_{ii'}[1 - \rho_j + \mathcal{L}^*(Jj\lambda i)](\varepsilon_j + w_{\lambda i}) \\
&\quad - \delta_{jj'}\delta_{\lambda\lambda'}\delta_{ii'}(1 + \mathcal{L}(Jj\lambda i))\frac{1}{4}\sum_{i_1}\mathcal{A}(\lambda i i_1)\mathcal{L}_{J|j}^*(j\lambda i|j\lambda i_1)
\end{aligned}$$

# Interaction strengths

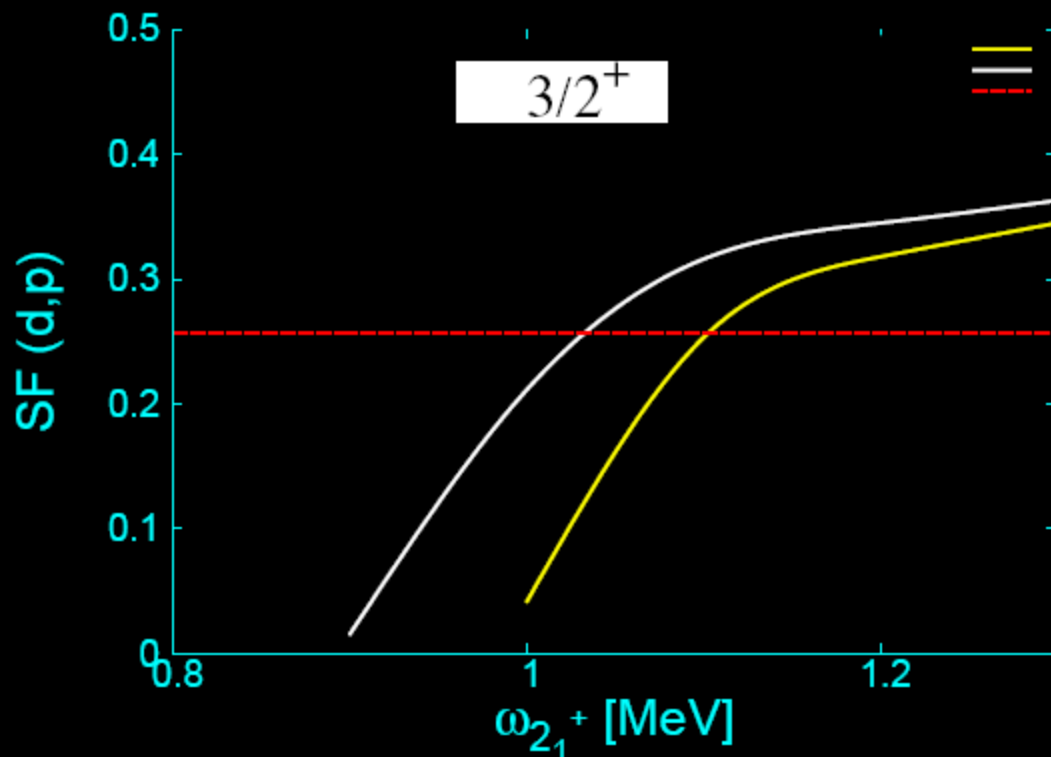
$^{131}\text{Ba}$



# Spectrum ( $^{131}\text{Ba}$ ) @ $\omega_{2_1^+} = 1 \text{ MeV}$



# Spectroscopic factors



Nuclide	Exp	$\omega_{2_1^+}$ , QPM_BCW_P	$\omega_{2_1^+}$ , EQPM_BCW_P
$^{123}\text{Te}$	0.5	1.4	1.3
$^{125}\text{Te}$	0.46	1.5	1.3
$^{127}\text{Te}$	0.38	1.5	1.35



# Transition probabilities in odd-even nuclei

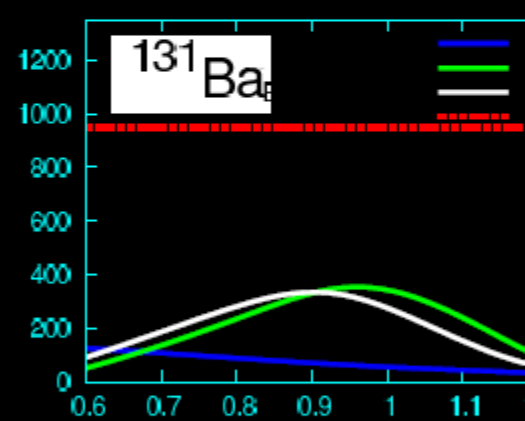
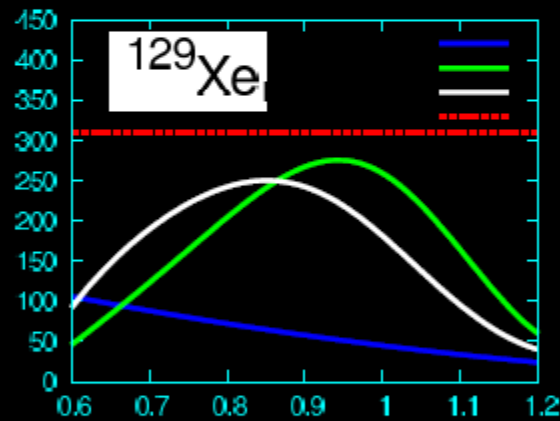
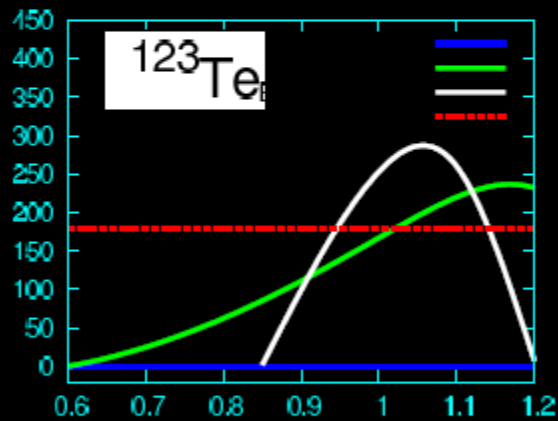
$$B(E\lambda; J_1\nu_1 \rightarrow J_2\nu_2) = \frac{1}{\pi^2_{J_1}} \left[ \sum_i U(J_1\nu_1 J_2\nu_2 \lambda i) \sqrt{B(E\lambda; g.s. \rightarrow \lambda_1)} \right]$$
$$\approx \frac{1}{\pi^2_{J_1}} U^2(J_1\nu_1 J_2\nu_2 \lambda 1) B_{\text{even}}(E\lambda; g.s. \rightarrow \lambda_1).$$

$$U(J_1\nu_1 J_2\nu_2 \lambda i) =$$

$$= \frac{\pi_{J_1}}{\pi_\lambda} [C_{J_2\nu_2} D_{J_2\lambda i}(J_1\nu_1) - E_{J_2\nu_2} F_{J_2\lambda i}(J_1\nu_1)] [1 + L(J_1 J_2 \lambda i)] +$$

$$(-)^{J_1 - J_2 + \lambda} \frac{\pi_{J_2}}{\pi_\lambda} [C_{J_1\nu_1} D_{J_1\lambda i}(J_2\nu_2) - E_{J_1\nu_1} F_{J_1\lambda i}(J_2\nu_2)] [1 + L(J_2 J_1 \lambda i)]$$

$$B(E2|3/2^+ \rightarrow 1/2^+)$$



# Summary

Renormalized qp-ph interaction strengths have been derived using the Extended Boson Approximation within a 1ph model

Some effects due to the qp-ph interaction weakening have been presented - spectroscopic factors, transition probabilities and spectra

The results of the numerical calculations indicate an overall improved agreement with the experiment

*Thank you for your attention!*

